In The Name Of GOD

Assignment #1

Mechanics of materials

97-98-1 Semester

- First read the sample problem.
- Answer the questions after the sample problem, based on approach that is used in sample problem.

- Note1: write your answers in A4 pages.
- Note2: Use both side of each paper.

- Give me your answers on 29th of October in the class. (7th of Aban)

- If there was any question here is my phone number: 09117061206

Good Luck

TABLE D/3 PROPERTIES OF PLANE FIGURES

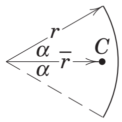
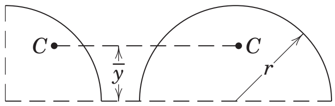
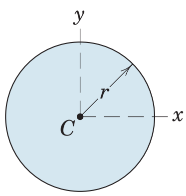
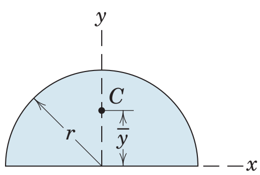
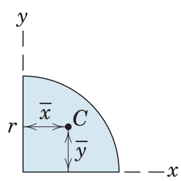
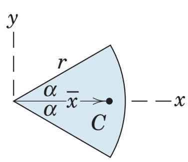
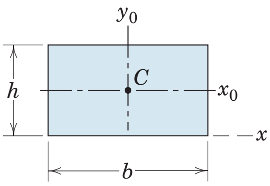
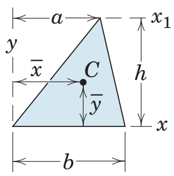
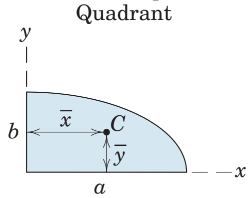
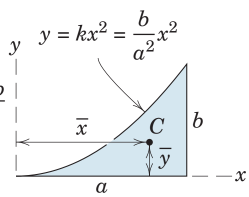
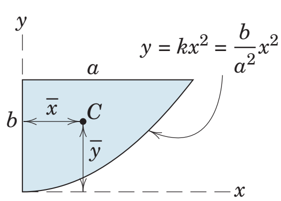
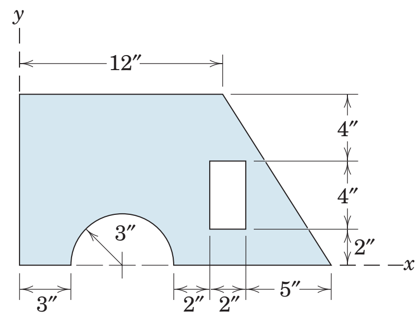
FIGURE	CENTROID	AREA MOMENTS OF INERTIA
Arc Segment 	$\bar{r} = \dfrac{r \sin \alpha}{\alpha}$	—
Quarter and Semicircular Arcs 	$\bar{y} = \dfrac{2r}{\pi}$	—
Circular Area 	—	$I_x = I_y = \dfrac{\pi r^4}{4}$ $I_z = \dfrac{\pi r^4}{2}$
Semicircular Area 	$\bar{y} = \dfrac{4r}{3\pi}$	$I_x = I_y = \dfrac{\pi r^4}{8}$ $\bar{I}_x = \left(\dfrac{\pi}{8} - \dfrac{8}{9\pi}\right) r^4$ $I_z = \dfrac{\pi r^4}{4}$
Quarter-Circular Area 	$\bar{x} = \bar{y} = \dfrac{4r}{3\pi}$	$I_x = I_y = \dfrac{\pi r^4}{16}$ $\bar{I}_x = \bar{I}_y = \left(\dfrac{\pi}{16} - \dfrac{4}{9\pi}\right) r^4$ $I_z = \dfrac{\pi r^4}{8}$
Area of Circular Sector 	$\bar{x} = \dfrac{2}{3} \dfrac{r \sin \alpha}{\alpha}$	$I_x = \dfrac{r^4}{4}(\alpha - \dfrac{1}{2} \sin 2\alpha)$ $I_y = \dfrac{r^4}{4}(\alpha + \dfrac{1}{2} \sin 2\alpha)$ $I_z = \dfrac{1}{2} r^4 \alpha$

TABLE D/3 PROPERTIES OF PLANE FIGURES *Continued*

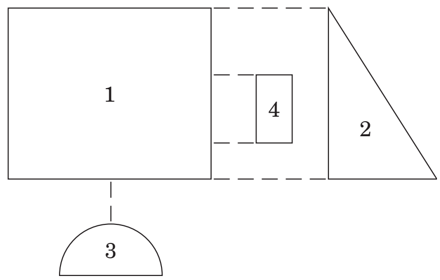
FIGURE	CENTROID	AREA MOMENTS OF INERTIA
Rectangular Area 	—	$I_x = \frac{bh^3}{3}$ $\bar{I}_x = \frac{bh^3}{12}$ $\bar{I}_z = \frac{bh}{12}(b^2 + h^2)$
Triangular Area 	$\bar{x} = \frac{a+b}{3}$ $\bar{y} = \frac{h}{3}$	$I_x = \frac{bh^3}{12}$ $\bar{I}_x = \frac{bh^3}{36}$ $I_{x_1} = \frac{bh^3}{4}$
Area of Elliptical Quadrant 	$\bar{x} = \frac{4a}{3\pi}$ $\bar{y} = \frac{4b}{3\pi}$	$I_x = \frac{\pi ab^3}{16}$, $\bar{I}_x = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)ab^3$ $I_y = \frac{\pi a^3 b}{16}$, $\bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)a^3 b$ $I_z = \frac{\pi ab}{16}(a^2 + b^2)$
Subparabolic Area Area $A = \frac{ab}{3}$ 	$\bar{x} = \frac{3a}{4}$ $\bar{y} = \frac{3b}{10}$	$I_x = \frac{ab^3}{21}$ $I_y = \frac{a^3 b}{5}$ $I_z = ab\left(\frac{a^3}{5} + \frac{b^2}{21}\right)$
Parabolic Area Area $A = \frac{2ab}{3}$ 	$\bar{x} = \frac{3a}{8}$ $\bar{y} = \frac{3b}{5}$	$I_x = \frac{2ab^3}{7}$ $I_y = \frac{2a^3 b}{15}$ $I_z = 2ab\left(\frac{a^2}{15} + \frac{b^2}{7}\right)$

SAMPLE PROBLEM

Locate the centroid of the shaded area.



Solution. The composite area is divided into the four elementary shapes shown in the lower figure. The centroid locations of all these shapes may be obtained from Table D/3. Note that the areas of the "holes" (parts 3 and 4) are taken as negative in the following table:



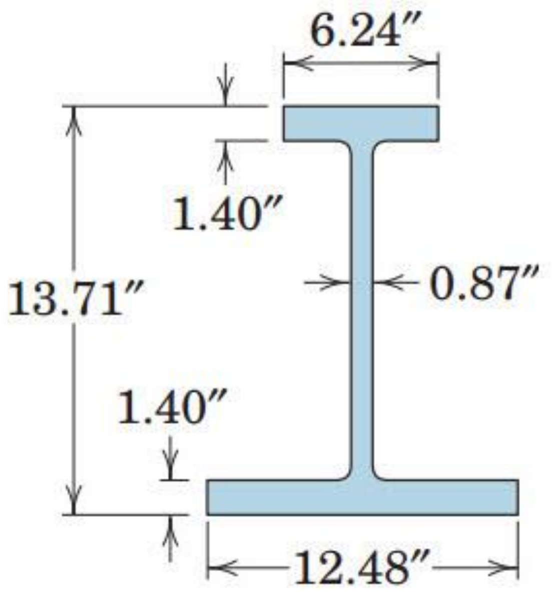
PART	A in.^{2}	$\bar{x}$ in.	$\bar{y}$ in.	$\bar{x}A$ in.^{3}	$\bar{y}A$ in.^{3}
1	120	6	5	720	600
2	30	14	10/3	420	100
3	−14.14	6	1.273	−84.8	−18
4	−8	12	4	−96	−32
TOTALS	127.9			959	650

The area counterparts to Eqs.

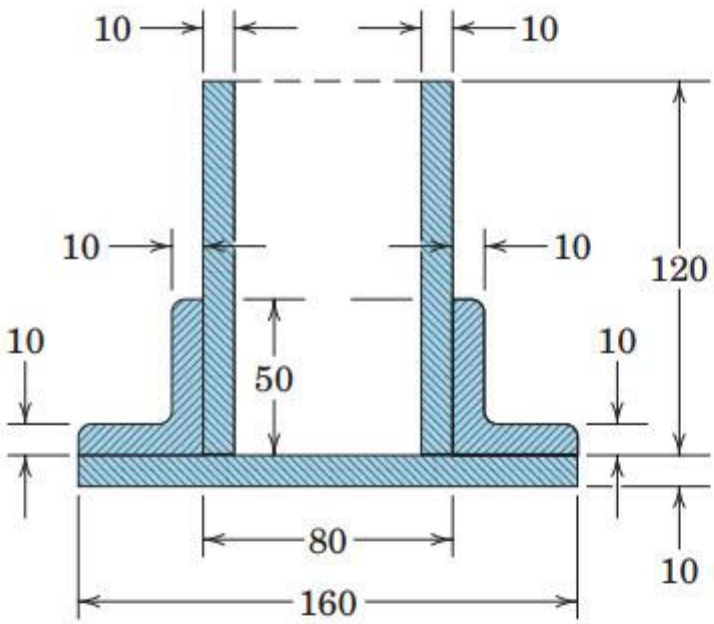
$$\left[\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A}\right] \qquad \bar{X} = \frac{959}{127.9} = 7.50 \text{ in.}$$ *Ans.*

$$\left[\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A}\right] \qquad \bar{Y} = \frac{650}{127.9} = 5.08 \text{ in.}$$ *Ans.*

Determine the height above the base of the centroid of the cross-sectional area of the beam. Neglect the fillets.

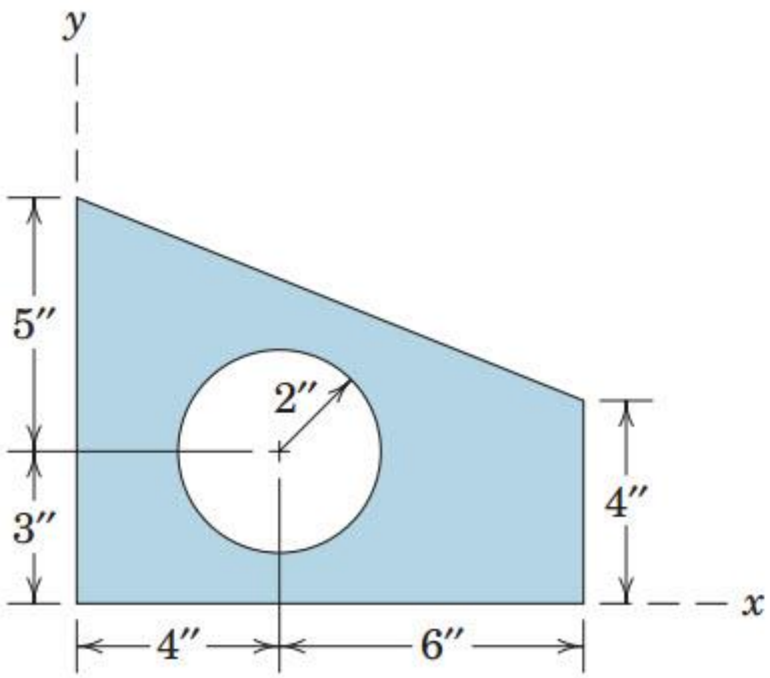


Determine the distance $\overline{H}$ from the bottom of the base plate to the centroid of the built-up structural section shown.



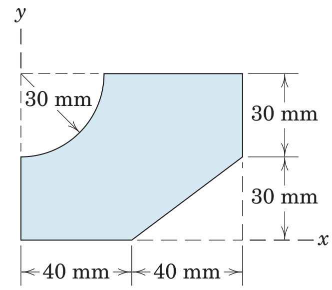
Dimensions in millimeters

Determine the x- and y-coordinates of the centroid of the shaded area.



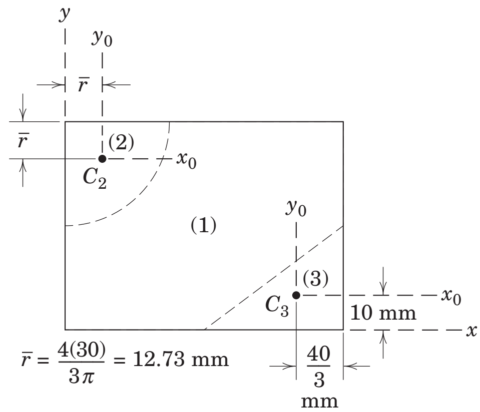
SAMPLE PROBLEM

Determine the moments of inertia about the x- and y-axes for the shaded area. Make direct use of the expressions given in Table D/3 for the centroidal moments of inertia of the constituent parts.



Solution. The given area is subdivided into the three subareas shown—a rectangular (1), a quarter-circular (2), and a triangular (3) area. Two of the subareas are "holes" with negative areas. Centroidal x_0–y_0 axes are shown for areas (2) and (3), and the locations of centroids C_2 and C_3 are from Table D/3.

The following table will facilitate the calculations.



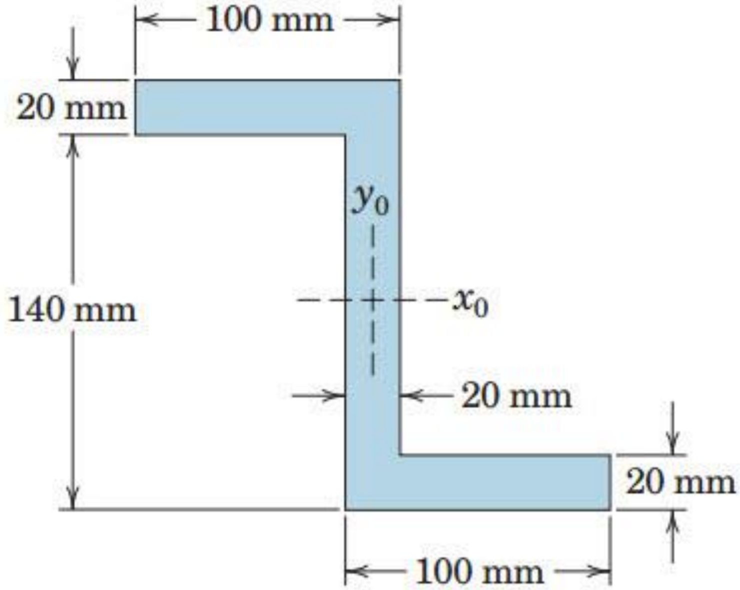
PART	A mm^2	d_x mm	d_y mm	Ad_x^2 mm^3	Ad_y^2 mm^3	$\bar{I}_x$ mm^4	$\bar{I}_y$ mm^4
1	80(60)	30	40	$4.32(10^6)$	$7.68(10^6)$	$\frac{1}{12}(80)(60)^3$	$\frac{1}{12}(60)(80)^3$
2	$-\frac{1}{4}\pi(30)^2$	(60 – 12.73)	12.73	$-1.579(10^6)$	$-0.1146(10^6)$	$-\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)30^4$	$-\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)30^4$
3	$-\frac{1}{2}(40)(30)$	$\frac{30}{3}$	$\left(80 - \frac{40}{3}\right)$	$-0.06(10^6)$	$-2.67(10^6)$	$-\frac{1}{36}\,40(30)^3$	$-\frac{1}{36}\,(30)(40)^3$
TOTALS	3490			$2.68(10^6)$	$4.90(10^6)$	$1.366(10^6)$	$2.46(10^6)$

$[I_x = \Sigma \bar{I}_x + \Sigma A d_x^{\,2}]$ $\quad I_x = 1.366(10^6) + 2.68(10^6) = 4.05(10^6)\text{ mm}^4$ *Ans.*

$[I_y = \Sigma \bar{I}_y + \Sigma A d_y^{\,2}]$ $\quad I_y = 2.46(10^6) + 4.90(10^6) = 7.36(10^6)\text{ mm}^4$ *Ans.*

The following sample problem will determine I_x by a different technique. For example, the area moment of inertia of subareas (1) and (3) about the x-axis are commonly tabulated quantities. While the above solution began with the centroidal moments of inertia of subareas (1) and (3), the following sample problem will make more direct use of the tabulated moments of inertia about the baselines.

Determine the moments of inertia of the Z-section about its centroidal x_0- and y_0-axes.



Calculate the product of inertia of the shaded area about the x-y axes. (*Hint:* Take advantage of the transfer-of-axes relations.)